



Examiners' Report
Principal Examiner Feedback

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Pearson Edexcel International A Level
In Pure Mathematics 4 (WMA14)

Paper: WMA14 / 01

General

This paper proved accessible to the candidates. The questions differentiated well, with most giving rise to a good spread of marks. There were questions and marks available to E grade candidates and there was also suitable material to challenge A grade candidates. Students find new topic of “proof” challenging and as indicated in the report on question 3, candidates often omit questions on this topic or struggle to adopt a suitable strategy to complete the proof. Students should become more confident in tackling questions on this topic as more past questions become available.

Report on individual questions

Question 1

This was a standard binomial expansion question and a significant proportion of candidates achieved full marks.

In part (a), the majority of candidates attempted to take out $\left(\frac{1}{4}\right)^{\frac{1}{2}}$ as a factor and correctly evaluated this as $\frac{1}{2}$. There were a few slips in evaluating the factor with $\frac{1}{16}$ or $4^{\frac{1}{2}}$ seen fairly frequently. The majority of candidates then used the correct structure for the binomial expansion and found the first four terms. A common incorrect “x” term used was $\frac{5x}{4}$. A very small minority of candidates unnecessarily found an additional term although this was not penalised in this part of the question. It was good to see that candidates understood “simplest form” includes signs and it was exceptionally rare that a candidate was penalised for leaving “+ -” instead of “-”. Some candidates attempted the direct expansion of $\left(\frac{1}{4} - 5x\right)^{\frac{1}{2}}$ and in the majority of cases this was unsuccessful with the candidates unable to evaluate coefficients such as “ ${}^{1/2}C_2$ ”. Very rarely, the Maclaurin series was applied. Despite this not being on the syllabus and full credit was given for this approach if negotiated correctly.

In part (b), the majority of candidates substituted $x = \frac{1}{100}$ into $\left(\frac{1}{4} - 5x\right)^{\frac{1}{2}}$ to get $\left(\frac{1}{5}\right)^{\frac{1}{2}}$ and substituted $x = \frac{1}{100}$ into their expansion; finally, either multiplying by 5 or taking the reciprocal of their evaluated expansion to get $\sqrt{5}$. A few candidates simply substituted for x in their expansion but did not relate this to $\sqrt{5}$. Those candidates who had given additional terms in part (a), lost the accuracy mark here in part (b). A very small minority of candidates unfortunately gave a decimal approximation for their answer and a very small minority of candidates tried to find a value for x by solving $\left(\frac{1}{4} - 5x\right)^{\frac{1}{2}} = \sqrt{5}$.

Question 2

Candidates were often successful on this question, with the majority of them scoring 4 out of the 5 marks.

Part (a) was a standard technique requiring the candidates to find the angle between two vectors. The majority of candidates successfully applied the scalar product. It was unclear whether candidates were finding angle BAC rather than the requested angle ABC or were just careless in not using $\overrightarrow{BA} \cdot \overrightarrow{BC}$ and used $\overrightarrow{AB} \cdot \overrightarrow{BC}$, leading to an acute angle. Both method marks were available to candidates in either case. Some candidates having used $\overrightarrow{AB} \cdot \overrightarrow{BC}$, then subtracted from 180° to find angle BAC, this was an acceptable approach. Unfortunately, many candidates gave the acute angle 67.35° as their final answer. There were very few slips with arithmetic or in taking inverse cosine following this approach. A small minority of candidates added the given vectors to find \overrightarrow{AC} and applied the cosine rule to triangle ABC. These candidates were usually successful though a few slips were made, most often with using 7^2 rather than 7 in the formula or again, for finding an incorrect angle. A very small minority of candidates appeared to be attempting to use vector product, which is acceptable. However, in almost all cases, it was simply a case of incorrect notation being used for the scalar product. The most common error with the unsuccessful candidates was in their misunderstanding of a “scalar” product and obtaining the vector $\begin{pmatrix} 12 \\ -10 \\ 24 \end{pmatrix}$ rather than the value of $12 \cdot 10 + 24$.

Candidates who gave their answer in radians were not able to gain the accuracy mark, nor were candidates giving their final answer to the nearest degree or just one decimal place.

Part (b) of this question required a simple application to find the area of a parallelogram using two adjacent side lengths and the angle between them, all of which had been found in part (a). The vast majority of candidates found the area of triangle ABC using $\frac{1}{2} |\overline{AB}| |\overline{BC}| \sin ABC$ and then doubled this. In a few scripts, candidates did this as a two-step operation and unfortunately rounded the area of the triangle to one decimal place before doubling, causing their final answer to be inaccurate. Candidates who had given the acute angle, BAC, in part (a) were able to earn full marks in part (b). Candidates who only found the area of triangle ABC lost both marks. Some candidates found a perpendicular height of the parallelogram and then proceeded to find the area. In these scripts, it was often the case that rounding errors caused the final accuracy mark to be lost. A very small minority of candidates incorrectly assumed that angle BAC was 90° and applied Pythagoras' theorem to find the height of the parallelogram.

Question 3

Although many candidates did not attempt this question, fully correct proofs were seen from those who were well prepared. The majority obtained the method mark for suggesting two appropriate odd numbers but full proofs with assumption, reason and conclusion were less common. False reasoning was sometimes seen, for example: " n is an integer so $n^2 + 1$ is odd". The most straightforward solutions assumed a greatest odd number n , and then went on to consider $n + 2$, but $2k + 1$ was a popular alternative starting point. Quite a few candidates simply listed odd numbers, 1, 3, 5, ..., $2k+1$, $2k+3$, $2k+5$ without comparing them.

Question 4

In part (a), most students were able to rearrange the equation to make t the subject successfully and substitute this into y . This way was chosen by the majority although a few rearranged the t from the y equation and substituted into x with limited success. The fractions were usually dealt with well but there were some instances where the algebraic manipulation was poor when attempting to simplify their expression. A small minority assumed the general form of the answer, substituted for x in terms of t and compared this to the expression for y but this strategy was often unsuccessful. Only very few gave a value for k possibly because they overlooked it. The main error was to assume that the denominator of the expression for y was zero, leading to $k = 5/3$.

Very few candidates achieved both marks part (b) although a good number gained one mark if they had some idea of what was required for the range. Candidates find the concept of domain and range difficult and it was clear that some confused range with domain and gave answers in terms of x rather than y .

Question 5

Candidates found this question particularly difficult. A surprising number of candidates could not get their derivative in terms of u successfully. They often had the correct derivative in terms of x but had difficulty establishing the correct expression in terms of u . There were also some who obtained an expression for dx/du but made an error when substituting so that their expression ended up in the numerator rather than the denominator. Unfortunately these were very costly errors and many only gained one mark in the whole question for use of appropriate limits. Those that obtained the correct derivative were usually able to integrate correctly and make progress to the right answer. A few made errors in the integration but those who differentiated and substituted correctly at the start often scored well.

Question 6

This question proved to be a very good source of marks for many candidates with many fully correct solutions seen. In part (a), only a few gained low marks usually from not applying the product rule on the RHS and/or the chain rule on the LHS. There were some careless errors in rearranging their expression with students miscopying their work from one line to the next or missing out terms between lines of working, although most students gained the method mark for collecting the dy/dx terms together. A small number of candidates chose to rearrange the given equation to make y the subject before differentiating, this was an acceptable approach; as was multiplying throughout by e^{2x} before attempting the differentiation.

Part (b) required candidates to use the information given about the curve, to find the co-ordinates of P, substitute these values for x and y into their $\frac{dy}{dx}$ to find the gradient of the tangent, take the negative reciprocal to find the gradient of the normal and then use this to obtain the equation of the normal at P. The majority of candidates followed this procedure carefully and achieved full marks. However, there was a surprisingly large number of candidates who simply took P to be the origin and lost all marks in this part. In some cases, the candidate found the correct co-ordinates for P but still used the point (0, 0) to evaluate the gradient. Substituting directly into their $\frac{-dx}{dy}$ was an acceptable approach to find the gradient of the normal and was seen from time to time.

Most candidates were able to use implicit differentiation to find $\frac{dy}{dx}$ in part (a) and in part (b), many of these were able to find the equation of the normal at point P gaining full marks.

Part (a) required candidates to apply the chain rule for differentiation to the y^2 term, which was done well by the vast majority, as was differentiating the term in x . It was good to see so many candidates cope with the differentiation of the product of y and e^{-2x} and to do this correctly in the majority of cases. Notation was accurate with just a small minority of candidates using y' for $\frac{dy}{dx}$ and even fewer candidates using the split derivative format ($8ydy + 3dx = -12ye^{-2x}dx + 6e^{-2x}dy$).

Question 7

Whilst most candidates attempted this question there were many who only gained between 1 and 3 marks. Having done an initial application of integration by parts, a significant number failed to make any further progress. Others, who did apply integration by parts twice, did not know how to collect the integrals together in order to make further progress. There were also a number of numerical mistakes and sign errors.

Students who scored full marks in part (a), had very little difficulty in gaining the two marks here. Unfortunately those that struggled in part a, were unable to substitute into an expression of the right format and gained no marks.

In part (ii), many candidates knew that they needed to divide by 3 and then take the square root although there were a significant number of cases where candidates failed to square root the "1/3". Of those who did reach this first stage correctly, many forgot to take the negative square root. Most but not all were then able to unravel the $2x + 10$ in the correct order for the second mark and this mark was available for candidates who failed to gain the first mark. The correct answer $x = 22.4$ was the most common one found

and $x = 30.3$ was a common incorrect answer from using $\frac{1}{3}$ instead of $\frac{1}{\sqrt{3}}$. Candidates who omitted the negative square root did not obtain a second solution. Some candidates spent a lot of time looking for additional solutions which were outside the range. For those candidates who did proceed correctly to obtain the second angle often rounded prematurely to obtain 57.7° rather than the required 57.6° .

Question 8

Many candidates did not fully understand the meaning of skew and only got as far as finding the values of μ and λ . Of those who found μ and λ (usually successfully), many went on to find $b = 7$ or use $b = 7$ to indicate consistent equations, and then just stated the lines were skew if $b \neq 7$. The majority, however, failed to state or show that the given lines were not parallel. A few candidates, having said that the lines did not intersect, went on to show that they were not perpendicular.

Question 9

This question required the candidates to find the volume of a solid of revolution from a curve given by parametric equations. It was done well by the majority of candidates with a good number gaining full marks. In part (a) almost all candidates were able to state that the volume of the solid of revolution about the x axis would be $\pi \int y^2 dx$. There were a lot of slips in working with y^2 with many candidates failing to show that they had used $\sin 2\theta = 2\sin\theta\cos\theta$. Moving from $(2\sin 2\theta)^2$ to $4\sin^2\theta\cos^2\theta$ or $8\sin^2\theta\cos^2\theta$ directly, did not earn the second method mark. A small number of candidates failed to find $\frac{dx}{d\theta}$ and incorrectly just replaced "dx" by "dθ". Some candidates incorrectly used $\tan^{-1}\sqrt{3} = 60^\circ$ for the upper limit and whilst this was condoned for the first mark, the final mark was withheld.

A few candidates mistakenly recalled the volume as $2\pi \int y^2 dx$; these were able to earn all but the final accuracy mark, as were those candidates who omitted the π .

A reasonable number of candidates followed the alternative method, replacing the $(2\sin 2\theta)^2$ in the numerator by $4(1 - \cos^2 2\theta)$ and replacing the $\sec^2\theta$ by a denominator of $\frac{1}{2}(1 + \cos 2\theta)$. Factorising the numerator which is a difference of two squares led to the result. This approach and the many variations of it were seen quite often and awarded the marks. A very small number of candidates having reached $16\pi \int \sin^2\theta d\theta$ then attempted to apply integration by parts. This did not lead the candidates to the required given form of the volume but it was an acceptable alternative and full marks were possible in part (b). A very small number of candidates tried to find the Cartesian equation of the curve $y = \frac{4x}{x^2+1}$ but were usually unsuccessful in reaching the given form of the volume.

In part (b), the majority of candidates gained the first two marks. Those candidates who had not been successful in part (a) could often make progress here by integrating the given form in part (a). There were rarely any errors in integrating or substituting. In the cases where there was an error, it was usually with the sign of the integration of the $\cos 2\theta$ term. A few candidates failed to integrate and simply substituted their limits of $\frac{\pi}{3}$ and 0 into their $8\pi(1 - \cos 2\theta)$. Once the correct volume had been seen, subsequent incorrect rearrangements of it were not penalised. A few candidates were penalised for using 60° instead of $\frac{\pi}{3}$ for θ , or for omitting π again in this part.

Question 10

In part (a) most candidates had a method to find partial fractions and obtained the first three marks. Just a few lost the final accuracy mark for not forming the fraction correctly despite having found the correct values for the constants. In part (b) most knew that they had to separate the variables and then integrate, which they usually managed successfully, although there were occasional errors in losing a negative sign. From that stage some candidates failed to introduce a constant of integration, scoring no further marks in this part. Those who did find the value of their constant sometimes had difficulty dealing with the logarithms and rearranging. The majority knew how to cross multiply and collect terms to get an expression for H , but there were some slips with brackets or signs. The given answer here persuaded many candidates to 'adjust' their working following obvious mistakes.

Part (c) was often well done despite failure in (b), but it was disappointing to see that some candidates had abandoned the question without attempting this part, which could have been done independently of (a) and (b).

Where part (d) was attempted, the most common answer was the correct one. Popular incorrect answers included 0, 3, 13 and ∞ .